compares this prediction with data from Ref. 13—this time for a wider sweep angle range of 60 to 75 deg. Although there is considerable scatter, the collapse is significantly better than for the standard (but wholly empirical) buffet frequency correlation parameter from Mabey,[18] which uses local semispan s and incidence $\sin\alpha$ to nondimensionalize the frequency. The comparison is particularly apt because Mabey's parameter was originally derived from the same data set.

Conclusions

This Note has presented a very simple conceptual model of the breakdown of a delta-wing vortex as a symmetry-breaking subcritical bifurcation from an axisymmetric unburst vortex to a helically symmetric translating spiral burst form. Applying continuity and rotational velocity constraints to the motion of a semi-infinite helix results in a critical swirl ratio similar to that given by conventional vortex stability analyses. The postburst helix is of opposite hand to the circulation of the vortex, with a pitch directly proportional to the vortex strength, and convects downstream at a rate less than freestream by a factor dependent on the helix inclination. The resultant velocity on the helix centerline is zero, given the stagnation point that is a characteristic feature of vortex breakdown. The model predictions agree reasonably well with measured helix pitch, inclination, radius, convection velocity, induced velocity, and fundamental frequency for burst vortex flow over delta wings with sweep angles from 60 to 75 deg. Given the major simplifications made in the initial derivation of the model, this agreement is very encouraging.

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Intensification of Aircraft Wake Vortices in Crosswind Shear

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Nomenclature

b	=	wing span
c, k	=	constants in Eq. (1)
x	=	spanwise (horizontal) coordinate
y	=	vertical coordinate
γ	=	nondimensional vortex density
τ	=	pseudotime

Introduction

ALTHOUGH wake turbulence has been blamed for deadly airline crashes in the past, it is rarely considered, under the current spacing standards, as a major safety problem. The public is led to believe that perhaps smaller aircraft may be at risk when flying into the persistent wakes of larger aircraft. The reality is such that, in the absence of credible scientific knowledge,[1] accidents/incidents due to wake turbulence may happen to any size aircraft, as a consequence of "unusual" combinations of operational and meteorological conditions.

The impact of the meteorological conditions on the decay of aircraft trailing vortices has been studied extensively.[2,3] However, much less attention has been paid to the fact that ambient conditions can also intensify aircraft vortices or make them decay at a slower rate. In such situations, an aircraft following another one may be subjected to larger rolling moments and turbulence than would normally be expected in the wake of the preceding aircraft. This Note explores such an example, namely, the "feeding" of a vortex on a crosswind shear. The phenomenon is consistent with the filamentation/consolidation principle (Chorin[4]), according to which the equally oriented vorticity tends to coalesce, producing localized concentrations of energy. Another explanation is possible using an analogy between fluid mechanics and magnetostatics.

Before proceeding further, note that there are two main types of interaction of aircraft trailing vortices and shear layers.[5] A weak interaction refers to a case where the vortex pair continues its descent, although possibly modified by the passage through the shear layer (change of slope, distance, etc.). The broken shear layer generates vortices of its own, but this multiple-vortex configuration is relatively benign. A strong interaction is the case when the shear layer is strong enough to capture the descending aircraft vortices and interact with them over extended periods of time. Potentially, this is

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a dangerous situation because vorticity contained in the shear layer may combine with the equally oriented aircraft vortex to produce a stronger vortex.

Following Schecter and Dubin,[6] the wing vortex will be termed prograde if it rotates with the shear and retrograde if it rotates against the shear.

Numerical Simulation

Simple demonstrations of the vortex intensification are given in two space dimensions using the convection of discrete vortex sheets. The thin vortex sheet concept, utilized by the method,[5] is a mathematical abstraction (support of the vorticity field) valid in the infinite Reynolds number limit. The local value of vortex density represents the discontinuity of the tangential velocity across the sheet. The numerical method discretizes the vortex sheet by constant γ segments. The self-induced motion is investigated in the length-dimensioned pseudotime τ, using a local conservation of vorticity; γ multiplied by segment length is equal to constant.

The first example concerns a straight-line sheet with the vortex density γ initialized as

$$\gamma = \left(k/\sqrt{\pi}\right)\exp(-k^2x^2) + c, \qquad -\infty < x < \infty \quad (1)$$

where k and c are constants. By selecting k, we set the maximum value of the Gaussian term; in the limit $k \to \infty$ it tends to the Dirac delta function. In our specific example, $k = 5$ is chosen, resulting in $\gamma_{\max} \approx 2.82 + c$. By selecting c, we set the background vorticity: $c = 0$ describes an isolated vortex, $c > 0$ a prograde vortex, and $c < 0$, $\gamma_{\max} > 0$ a retrograde vortex.

Figure 1 shows the results at time $\tau = 3$ (unit-length dimension) for $c = 0, 0.5$, and -0.5, satisfying the criteria for the three types of flow. In Fig. 1a, we observe an isolated vortex nearing a fully developed stage; there is no more vorticity left to be rolled up. At the same time, the prograde vortex in Fig. 1b keeps growing by attracting and winding up the surrounding line of vorticity. This rollup process can, in theory, continue indefinitely. In contrast, the retrograde vortex in Fig. 1c repudiates the neighboring (oppositely signed) shear. To distinguish between the positive and negative vorticity, segments whose $|\gamma| < 0.01$ are not plotted. The "torn" ends of the shear layer are seen to develop vortices of their own.

The second example, shown in Fig. 2, models a strong interaction of aircraft vortices with crosswind shear. The initial conditions are as follows.

Wing trailing sheet (elliptic loading):

$$\gamma = 4(2x/b)[1 - (2x/b)^2]^{-\frac{1}{2}}$$

$$-\tfrac{1}{2} < x/b < \tfrac{1}{2}, \quad y/b = 0 \quad (2)$$

Crosswind shear

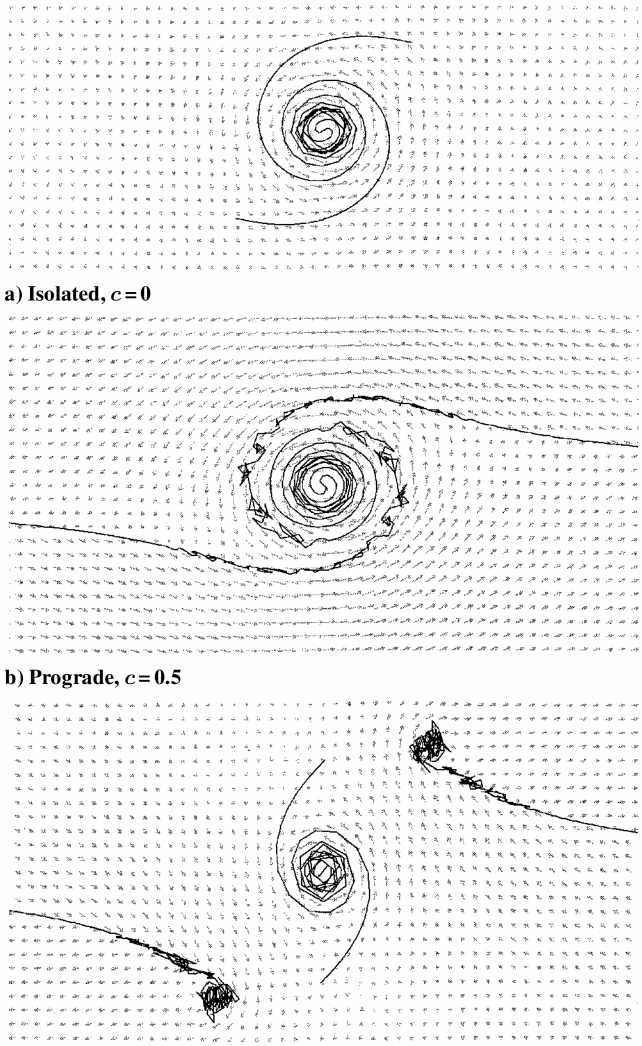
$$\gamma = -1.0, \qquad -\infty < x/b < \infty, \qquad y/b = -0.5 \quad (3)$$

The averages of the absolute values of the densities described by Eqs. (2) and (3) compare as

$$\frac{1}{b}\int_{-b/2}^{b/2} |\gamma(x)|\,\mathrm{d}x = \begin{cases} 4, & \text{wing} \\ 1, & \text{shear} \end{cases}$$

On the aircraft trailing sheet, segments of length varying like a cosine of a constant angular increment are used. This distribution provides a more dense segmenting near wing tips where $|\gamma|$ is large; compare Eq. (2). On the crosswind shear layer, segments of equal length are used on a central interval near the aircraft, and segments of length varying like a tangent of a constant angular increment are used on the left and right. Highly stretched segments, $|\gamma| < 0.02$, are not plotted, although they are still included in the computation.

Figure 2a ($\tau/b = 0$) shows the initial velocity field. A negative γ rotates flow clockwise, that is, above the shear layer the flow is to the right and below the shear layer to the left. The aircraft trailing sheet perturbs the sectionally uniform flowfield, induced by the constant-density shear layer, both above and below.



a) Isolated, $c = 0$

b) Prograde, $c = 0.5$

c) Retrograde, $c = -0.5$

Fig. 1 Vortex at $\tau = 3$, initialized by Eq. (1), $k = 5$.

Figure 2b ($\tau/b = 1$) shows that the rollup of the aircraft vortex sheet is underway. The vortices travel along oblique descent paths into the crosswind shear layer, indenting it and inducing secondary vortices.

Figure 2c ($\tau/b = 2$) shows a further stage of this development. On the left, the prograde vortex and its shear layer counterpart rotate about each other in the clockwise direction (both are negative). At the same time, more and more of the (Helmholtz-unstable) shear layer is drawn into the orbit of the vortex. The retrograde vortex on the right is seen to move upward with its oppositely signed shear layer counterpart. A similar vortex rise has been reported earlier by Proctor[7] and Zheng and Baek.[8]

Figure 2d ($\tau/b = 4$) shows that the prograde vortex is already surrounded by the (equally signed) shear layer. The circulation of this system is larger than that of the original wing tip vortex. Because of a relatively coarse segmenting, the structure of the shear layer vortices has disappeared, but the intensification of the prograde vortex has, nevertheless, been demonstrated. The retrograde vortex, on the other hand, does not attract the (oppositely signed) vorticity and does not change its strength significantly. The neighboring shear layer vortices continue to distance themselves and to roll up on their own.

A strong interaction of the aircraft pair of vortices with the shear layer, thus, may result in a large-scale, multivortex configuration. However, the individual flow patterns shown earlier in Fig. 1 are easily identified.

Admittedly, a simple convection of thin vortex sheets may not reflect exactly on the complex behavior of aircraft vortices in variable atmospheric conditions. In particular, an actual shear layer contains

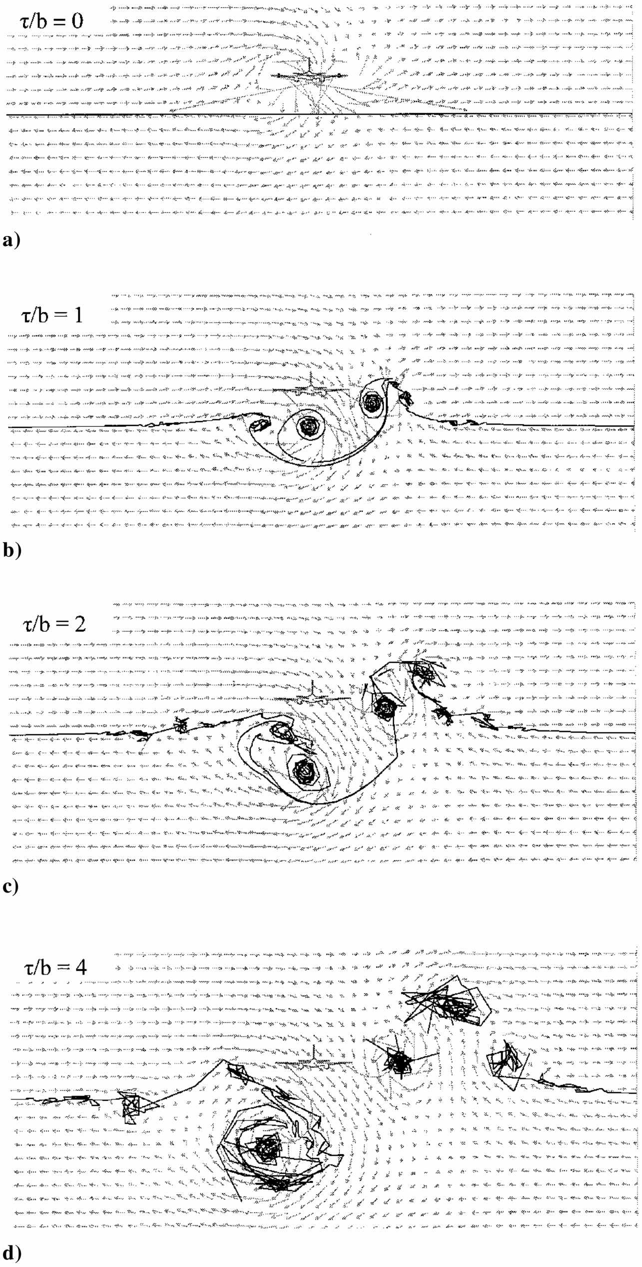


Fig. 2 Interaction of vortex pair and crosswind shear, initialized by Eqs. (2) and (3).

active turbulence, which tends to destabilize the vortex pair. On the other hand, convection is known to be the dominant factor in early stages of the vortex development, and flow patterns similar to those shown here have been found repeatedly, more recently in modeling the interactions with thick shear layers.[9]

Conclusions

Based on the convection of thin vortex sheets, it has been shown that aircraft vortices can temporarily intensify in the presence of crosswind shear. An aircraft following another one may, thus, be subjected to a stronger vortex than that expected in the wake of the leading aircraft in a quiescent atmosphere.

Although wake turbulence measurements may soon become a part of the aircraft airworthiness process, it is believed that still greater safety of air travel is achievable by minimizing the vortex intensification in wind shear and by avoiding wake vortex encounters as much as possible. This may involve guiding the aircraft to turn into crosswind only when sufficiently distant from the airports and equipping them, individually, with wind shear detectors, wake vortex sensors, and other suitable warning devices.

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Effect of Leading-Edge Cross-Sectional Shape on Nonslender Wing Rock

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Introduction

RECENT experimental results,[1] obtained in low-speed wind-tunnel tests, showed that the high-alpha rolling moment characteristics of a 45-deg delta wing are very sensitive to the cross-sectional shape of the leading edge, especially in regard to the unsteady aerodynamics. Figure 1 shows the geometric differences between the two tested delta wing models. The 9.1% thick wing had a semicircular leading-edge cross section, whereas the thin (1.25% thick) wing had a sharp-edged, 20-deg double bevel. As expected, the three-dimensional stall, realized when vortex breakdown reaches the apex,[2] occurs earliest on the sharp-edged delta wing (Fig. 2). For both wings, leading-edge flow separation with associated vortex shedding is delayed by an angle of attack $\Delta\alpha_v$, where $\Delta\alpha_v$ is determined by the leading-edge cross section as follows[3]:

$$\Delta\alpha_v = \tan^{-1}(\tan\alpha_{\mathrm{LE}}\cos\Lambda) \tag{1}$$

where α_{LE} is the crossflow angle of attack (normal to the leading edge) that has to be exceeded before crossflow separation occurs and a leading-edge vortex starts being generated. For the sharp-edged wing, $\alpha_{\mathrm{LE}} = 10$ deg. For the thick wing, α_{LE} is determined by the nose roundness[4] (Fig. 3). According to the experimental airfoil results in Fig. 3, one expects that $\alpha_{\mathrm{LE}} \geq 16$ deg for the thick wing. With $\Lambda = 45$ deg, these values of α_{LE} give, in Eq. (1), $\alpha_v = 7.1$ and $\alpha_v \geq 11.4$ deg for the thin and thick wing, respectively. The $\Delta\alpha_v \geq 4.3$ deg difference is in basic agreement with the experimental results in Fig. 2, showing maximum lift to occur at $\alpha \approx 20$ and 25 deg, respectively, for the thin and thick wing. This implies that the breakdown progresses with increasing $\alpha_{\mathrm{eff}} = \alpha - \alpha_v$ at roughly the same rate in the two cases.

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